What is Time?

Jos Stam

Autodesk Research
University of Toronto

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Menu Prix Fixe

Confit de Nucleus a la sauce Maya

Escaloppe Temporelle cuite avec une Gratine a la sauce Canadienne

Un Flambe a l'Optimisazation parseme avec des nombres duels

Vin de Table: The Art of Fluid Animation. Grand Cru Chinois.
Entree
Entree

Nucleus: 10 years after...
## Nucleus World Tour

<table>
<thead>
<tr>
<th>City</th>
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<td>Vienna</td>
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<td>Lyon</td>
<td>Apr 2017</td>
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Ruysdael (1628-1682)
Nucleus

MAYA

~ 100 files

~ 40,000 files
Used in Movies

Weta FX, Wellington, New Zealand
nHair Examples

Thanks Ken Taki!
Duncan’s Corner

http://area.autodesk.com/blogs/duncan
Slinky
Paper Airplanes
The Brain
nParticles and Fluid Effects
Main Course
Main Course

What is Time?

... tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic, toc, tic ...

Au prochain top il sera exactement...
St. Augustine

348-430 AD
Time Does not Exist

*Time is a fiction created by the mind.*
Infinite Linear Time
Time is finite

My ugly left hand
Time is finite

Our ugly Universe
Time is Circular
Time is Circular
“Zen”

But there is a direction!
A Simple Spring
Fourier Series

\[ f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i \frac{2\pi nt}{T}} \]

Any function is a sum of ever oscillating ideal springs
Finnegans Wake 1939 (17 Years)

James Joyce, 1882 – 1941.

“If you met on the binge a poor acheseyeld from Ailing..."
Finnegans Wake

Riverrun, past Eve and Adam's, from swerve of shore to bend of bay, brings us by a commodius vicus of recirculation back to Howth Castle and Environs.

Lots of dense prose and finally:

A way a lone a last a loved a long the

A way a lone a last a loved a long the / riverrun, past Eve and Adam's, from swerve of shore to bend of bay, brings us by a commodius vicus of recirculation back to Howth Castle and Environs.
Mulholland Dr. (2001)

No End of the movie
Memento (2000)

End of the movie
Pulp Fiction (1994)

End of the movie?
Fundamental Physics: Time has no Direction

*Laws are invariant under the transformation*

Time \rightarrow \textit{minus} Time
How do we deal with this?
How do we deal with this?

But suppose that in every airport we were to remove all the signs and flight information, while maintaining exactly the same number of flights, as shown in Figure 11(b). A person starting from Easthampton would still arrive in New York, since that is the only airport connected to Easthampton. However, without the signs to guide him, it would be very difficult for him to pick out the return flight to Easthampton from the many gates in the New York airport. The plane he gets on may be headed for San Francisco. If, in San Francisco, he then tries another plane again without any guidance, he could perhaps arrive in Tokyo. If he keeps on going this way, his chance of getting back to Easthampton is very slim indeed. In this example, we see that microscopic reversibility is strictly maintained. When all the airport destination signs and other flight information are given clearly, then macroscopically we also have reversibility. On the other hand, if all such information is withheld, then the whole macroscopic process appears irreversible. Thus, macroscopic irreversibility is not in conflict with microscopic reversibility.

Tsung-Dao (T. D.) Lee 李政道
How do we deal with this?
time is Like Temperature

\[ T = \text{sum of } \text{squared} \]
time is an emergent phenomenon

dt = sqrt( sum of O→O squared / energy )

Time emerges from changes in positions!
Nucleus & Position-Based Dynamics
The concept of one event happening before another in a distributed system is examined, and is shown to define a partial ordering of the events. A distributed algorithm is given for synchronizing a system of logical clocks which can be used to totally order the events. The use of the total ordering is illustrated with a method for solving synchronization problems. The algorithm is then specialized for synchronizing physical clocks, and a bound is derived on how far out of synchrony the clocks can become.

Key Words and Phrases: distributed systems, computer networks, clock synchronization, multiprocess systems

CR Categories: 4.32, 5.29
Logical Time

A network diagram showing the relationships between different events labeled as e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13, and e14. The arrows indicate the order and dependencies among the events.
A Taste of Barbados in February

Thanks Paul Kry!
Doing Math *without* Math
Writing Optimization Code

*Without*

Continuous Math
“A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street.”

David Hilbert
Ou alors, on pourrait agir ainsi : tu irais à un gala nippon.
Il y aurait pour ton grand plaisir, car on sait ton goût pour l'art subtil du Go, un naïf affrontant dans un match amical un champion, un « Kan Shu », sinon un « Kudan » : Kaku Takagawa, mais disposant, pour adoucir la disproportion, d'un fort handicap, non d'un « furin » mais d'un « Naka yotsu ». Kaku Takagawa ouvrirait par un « Moku hadzushi » ; son opposant s'absorberait dans un « Ji dori Go » aussi maladroit...

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Examples of optimization
In Nature it occurs naturally
Statics: Geodesics

15 and a half hour flight
Dynamics: Alembert, Hertz and Einstein

Dynamics is a geodesic in space-time
Eisenhart metric

Dynamics is a geodesic in space-time + one extra dimension

Concept of lifting.
Solve a problem in a higher dimensional space where there is more freedom and then project back.
optimization

code

controls

output

optimizer
optimization
optimization

code

controls

d..doutput

ddoutput

doutput

output

optimizer
Back to basics: one dimension
One dimension: gradient: d
One dimension: Hessian: \( dd \)
Optimization: *let stick to* differential

- code
- controls
- optimizer
- output
- doutput
Optimization: *cannon balls in 2D*
Gradient Based Optimizer

\[ u[N] \rightarrow \text{code} \rightarrow f \rightarrow \text{optimizer} \rightarrow u_{\text{new}}[N] \]

\[ u[N] \rightarrow \text{dcode} \rightarrow df[N] \]
We Need a Discrete Gradient!

*Usually*: finite differences, finite elements, etc.

*Instead*:

Differentiate code *at the* code level
Example

```c
float optim_func ( float u[2] )
{
    float s = u[0]*cosf(2*u[1]);
    float t = u[0]*u[0]*u[1];
    float f = s*s+t*t;
    return ( f );
}
```
Hyper-Numbers

\[ z = a + i \ b \]

\[ i^2 = -1 \] Complex numbers (“Awesome Numbers”)

\[ i^2 = 1 \] Hyperbolic numbers (Special Relativity)

\[ i^2 = 0 \] Dual numbers (Automatic Differentiation)
Dual-Numbers

“Number theoretic phase space”

\[ z = f + i \, df \quad \quad i^2 = 0 \]

With these numbers you compute the function and the differential at the same time!

**Good for Optimization!**

Also works for higher differentials btw... Hyper-Dual Numbers...
Dual-Numbers encode Calculus

\[(f + i \, df) + (g + i \, dg) = (f + g) + i \, (df + dg)\]

\[(f + i \, df) \times (g + i \, dg) = (f \times g) + i \, (df \times g + f \times dg) \quad i^2 = 0\]

Etc.

Automatic Differentiation: AD.h

template <int N> class dfloat
{
public:

    float v[N+1];

    // implementation
}

dfloat<2> u[2], x, f;
Automatic Differentiation: AD

template <int N> class dfloat
{
public:

    float v[N+1];

dfloat (){
    for ( int i=0 ; i<=N ; i++ ) v[i] = 0.0f;
}
dfloat ( float s ){
    v[0] = s;
    for ( int i=1 ; i<=N ; i++ ) v[i] = 0.0f;
}
}

dfloat<2> u[2], x(0.05f), f(3.14f);
Automatic Differentiation: AD

template <int N> class dfloat
{
public:

    float v[N+1];

dfloat & operator = ( const dfloat & a ){
    for ( int i=0 ; i<=N ; i++ ) v[i] = a.v[i];
    return ( *this );
}
dfloat & operator = ( const float s ){
    v[0] = s;
    for ( int i=1 ; i<=N ; i++ ) v[i] = 0.0f;
    return ( *this );
}
void val ( int i, float s ){
    v[i] = s;
}
}

dfloat<2> u, x, f(0.0001f);
u[0].val(1,1.0f);
u[1].val(2,1.0f);
x = 2.03f;
f = x;
Automatic Differentiation: AD

dfloat<2> u, x(0.45f), f(0.0001f);

u[0].val(1,1.0f);
u[1].val(2,1.0f);
x = u*u + f;
f = x/u*f;
Example

```c
static float optim_func ( float x, float y )
{
    float s = x * cosf(2*y);
    float t = x * x * y;
    float f = s*s+t*t;
    return ( f );
}
```
Example

static float optim_func ( float x, float y )
{
    float s = x * cosf(2*y);
    float t = x * x * y;
    float f = s*s + t*t;
    return ( f );
}

static dfloat<2> doptim_func ( dfloat<2> x, dfloat<2> y )
{
    dfloat<2> s = x * dcos(2*y);
    dfloat<2> t = x * x * y;
    dfloat<2> f = s*s + t*t;

    return ( f );
}
Example

```cpp
static dfloat<2> doptim_func ( dfloat<2> x, dfloat<2> y )
{
    dfloat<2> s = x * dcos(2*y);
    dfloat<2> t = x * x * y;
    dfloat<2> f = s*s + t*t;

    return ( f );
}
```
Fun Example: Rigid Bodies

fast

slow
Three-Level Optimization

1. Meta parameters
2. Global Search
3. Local Optimization
Three-Level Optimization

信达雅

— Yan Fu (1853-1921)

Faithfulness  Fluency  Elegance
Local Optimization: BFGS
Local Optimization: BFGS

optimum
Global Search: MCMC

optimum
Global Search: MCMC
What is BFGS?

Broyden, Fletcher, Goldfarb and Shanno
No Really what is BFGS?

Algorithm 2.2 BFGS algorithm

Input: $t \leftarrow 0$, $\theta_0 \leftarrow \text{rand}(0, 1)$, $g_0 \leftarrow g(\theta_0)$, $D_0 \leftarrow H(\theta_0)^{-1}$;

Output: $\theta^{t+1}$ minimize cost function $L_e(\theta)$;

1: while $\|g_t\| > \varepsilon$ do
2:     $d_t \leftarrow -D_t g_t$; # compute search direction
3:     $\lambda_t \leftarrow \arg\min_{\lambda > 0} L_e(\theta_t + \lambda d_t)$; # line search meeting Wolfe conditions
4:     $s_t \leftarrow \lambda_t d_t$, $\theta_{t+1} \leftarrow \theta_t + s_t$; # update parameters
5:     $g_{t+1} \leftarrow g(\theta_{t+1})$, $y_t \leftarrow g_{t+1} - g_t$;
6:     $D_{t+1} \leftarrow \left[ I - (y_t s_t)^{-1} s_t y_t^T \right] D_t \left[ I - (y_t s_t)^{-1} y_t s_t^T \right] + (y_t s_t)^{-1} s_t s_t^T$;
7:     $t \leftarrow t + 1$;
8: end while
What is MCMC?

Markov Chain Monte Carlo

Diagram showing a Markov chain with transitions between states:
- From sunny to cloudy with probability 0.3
- From sunny to rainy with probability 0.5
- From cloudy to sunny with probability 0.6
- From cloudy to rainy with probability 0.2
- From rainy to sunny with probability 0.4
- From rainy to cloudy with probability 0.1
- From cloudy to cloudy with probability 0.1
- From rainy to rainy with probability 0.5

Photo of an ornate room, possibly a casino, with a person and another person standing in the background.
Related to Subdivision Surfaces

- Limit probability distribution
- Probabilities add to one
- Transition matrix
- eigenvalues

- Limit surface
- Affine invariance
- Subdivision matrix
- eigenvalues

*Iterations of an operator reduced to powers of scalars!*
Hybrid-MCMC

Use the Gradient!
Hybrid-MCMC
Hybrid-MCMC

accept
Hybrid-MCMC

Possibly accept
Hybrid-MCMC

Possibly accept
Hybrid-MCMC
Hybrid-MCMC

We got a winner!
Hybrid-MCMC

BFGS will finish the job
Experiments: Validation

Goldstein-Price function
static dfloat<2> f_func ( dfloat<2> x, dfloat<2> y )
{
    dfloat<2> z;

    z = ( 1.0f + dsqr(x+y+1.0f) * (19.0f-14.0f*x+3.0f*dsqr(x)) ) *
    ( 30.0f + dsqr(2.0f*x-3.0f*y) *
    (18.0f-32.0f*x+12.0f*dsqr(x)+48.0f*y-36.0f*x*y+27.0f*dsqr(y)) );

    return ( z );
}
Experiments: Validation
Shape Optimization
Discussion

It is great to prototype and good for small problems

AD is not efficient for large problems

But it is still cool!
The Adjoint Method to the Rescue
Vin de Table
In Pin Yin: “you shi dan” : 尤 = kind of sounds like “Jos” 士 = “respected scholar” 丹 = “red.”
Merci!

a suivre...